11) a) $A = \begin{bmatrix} -1 & -2 \\ 0 & -3 \end{bmatrix}$, $Y' = AY$, $Y(0) = Y_0$

$$P(\lambda) = \det\left(\begin{bmatrix} \lambda+1 & 2 \\ 0 & \lambda+3 \end{bmatrix}\right) = \lambda^2 + 4\lambda + 3$$

Autoval. $\rightarrow \underbrace{\lambda^2 + 4\lambda + 3 = 0}_{\frac{-4 \pm \sqrt{16-12}}{2}}$ $\rightarrow$ $\boxed{\lambda_1 = -1 \quad \lambda_2 = -3}$

Para $\lambda = -1$

$$\begin{pmatrix} 0 & 2 \\ 0 & 2 \end{pmatrix} \rightarrow F2 \rightarrow F1 - F2 \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix} \rightarrow y = 0 \rightarrow \bar{X} = x \cdot \underbrace{(1,0)}_{\text{AUTOVECT. } \lambda = -1}$$

Para $\lambda = -3$

$$\begin{pmatrix} -2 & 2 \\ 0 & 0 \end{pmatrix} \rightarrow -2x + 2y = 0 \rightarrow x = y \rightarrow \bar{X} = x \cdot \underbrace{(1,1)}_{\text{AUTOVECT. } \lambda = -3}$$

$$Y(t) = k_1 \cdot e^{-t} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + k_2 \cdot e^{-3t} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Como $Y(0) = Y_0$: $Y_0 = (Y_a, Y_b)$

$$Y(t) = k_1 \cdot \begin{bmatrix} e^{-t} \\ 0 \end{bmatrix} + k_2 \cdot \begin{bmatrix} e^{-3t} \\ e^{-3t} \end{bmatrix}$$

$$Y(0) = k_1 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + k_2 \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} Y_a \\ Y_b \end{bmatrix}$$

$\rightarrow k_1 + k_2 = Y_a \rightarrow k_1 = Y_a - Y_b$.

$k_2 = Y_b$

SOLUCION A PVI:

$$\rightarrow \boxed{Y(t) = (Y_a - Y_b) \cdot \begin{bmatrix} e^{-t} \\ 0 \end{bmatrix} + Y_b \cdot \begin{bmatrix} e^{-3t} \\ e^{-3t} \end{bmatrix}}$$

6) $A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix}$ $\quad P(\lambda) = \det\left(\begin{bmatrix} \lambda-1 & 2 \\ 0 & \lambda-3 \end{bmatrix}\right) = \lambda^2 - 4\lambda + 3$

Autoval. $\to$ $\underbrace{\lambda^2 - 4\lambda + 3 = 0}_{\frac{4 \pm \sqrt{16-12}}{2}}$ $\to \lambda_1 = 3$

$\searrow \lambda_2 = 1$

Autovect.:

$\underline{\text{para } \lambda = 3}$

$\begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix} \to x = -y \to \bar{x} = y \cdot \underbrace{(-1, 1)}_{\substack{\text{AUTOVECT.} \\ \lambda = 3}}$

$\underline{\text{para } \lambda = 1}$

$\begin{pmatrix} 0 & 2 \\ 0 & -2 \end{pmatrix} \xrightarrow{F2 \to F1+F2} \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$ $\quad y = 0 \to \bar{x} = x \cdot \underbrace{(1, 0)}_{\substack{\text{AUTOVECT.} \\ \lambda = 1.}}$

$$Y(t) = k_1 . e^{3t} . \begin{bmatrix} -1 \\ 1 \end{bmatrix} + k_2 . e^{t} . \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Como $Y(0) = Y_0 = (Y_a, Y_b)$

$$k_1 . \begin{bmatrix} -1 \\ 1 \end{bmatrix} + k_2 . \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} Y_a \\ Y_b \end{bmatrix}$$

$$\to \begin{cases} -k_1 + k_2 = Y_a \to k_2 = Y_a + Y_b . \\ k_1 = Y_b \end{cases}$$

solución PVI:

$$\boxed{Y(t) = Y_b \begin{bmatrix} -e^{3t} \\ e^{3t} \end{bmatrix} + (Y_a + Y_b) . \begin{bmatrix} e^{t} \\ 0 \end{bmatrix}}$$

c) $A = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$, $p(\lambda) = \lambda^2 + 1$

Autoval: $\rightarrow$ $\lambda^2 + 1 = 0$ $\rightarrow \lambda_1 = i$

$\rightarrow \lambda_2 = -i$

Para $\lambda = i$

$$\begin{pmatrix} i-1 & 2 \\ -1 & i+1 \end{pmatrix} \begin{cases} (i-1)x + 2y = 0 \to ⊛ \\ -x + (i+1)y = 0 \to x = (i+1)y \end{cases}$$

$$⊛ \to (i-1).(i+1)y + 2y = 0 \to (i^2-1)y + 2y = 0 \to -2y + 2y = 0$$

$$\bar{X} = ((i+1)y, y) = y.\underbrace{(i+1;1)}_{\substack{\text{AUTOVECTOR} \\ \lambda = i}}$$

Para $\lambda = -i$ será el conjugado:

$$\boxed{\text{Autovector } \lambda = -i \to (i-1;1)}$$

$$\underbrace{\phi_1 = e^{ix}.\begin{pmatrix} i+1 \\ 1 \end{pmatrix}}, \quad \phi_2 = e^{-ix}.\begin{pmatrix} i-1 \\ 1 \end{pmatrix}$$

$$\phi_1 = [e^{0t}.\cos(t) + ie^{0t}\operatorname{sen}(t)].\begin{pmatrix} i+1 \\ 1 \end{pmatrix}$$

$$\phi_1 = [\cos(t) + i\operatorname{sen}(t)].\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \underbrace{\begin{pmatrix} i \\ 0 \end{pmatrix}}_{i\begin{pmatrix} 1 \\ 0 \end{pmatrix}}\right]$$

$$\to \phi_1 = \underbrace{\left[\cos t.\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \operatorname{sen} t.\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]}_{\text{PARTE REAL.}} + i\underbrace{\left[\cos t.\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \operatorname{sen} t.\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right]}_{\text{PARTE IMAGINARIA.}}$$

como $\phi_2 = \overline{\phi_1} \to \phi_1 + \phi_2 = 2Re(\phi_1)$ también son soluciones del homogéneo

$$\to \phi_1 - \phi_2 = 2i\, Im(\phi_1).\frac{1}{2i}$$

para q' quede todo en reales.

$$Re(\phi_1) = \cos t \cdot \begin{pmatrix}1\\1\end{pmatrix} - \text{sen}\,t \cdot \begin{pmatrix}1\\0\end{pmatrix}$$

$$Im(\phi_1) \cdot \frac{1}{2i} = \cos t \cdot \begin{pmatrix}1\\0\end{pmatrix} + \text{sen}\,t \cdot \begin{pmatrix}1\\1\end{pmatrix}$$

$$\rightarrow \; Z_1 = \begin{pmatrix}\cos t - \text{sen}\,t\\ \cos t\end{pmatrix}, \quad Z_2 = \begin{pmatrix}\cos t + \text{sen}\,t\\ \text{sen}\,t\end{pmatrix}$$

$$\rightarrow \; Y(t) = k_1 \cdot \begin{pmatrix}\cos t - \text{sen}\,t\\ \cos t\end{pmatrix} + k_2 \cdot \begin{pmatrix}\cos t + \text{sen}\,t\\ \text{sen}\,t\end{pmatrix}, \quad k_1, k_2 \in \mathbb{R}.$$

Como $Y(0) = Y_0 = (Y_a, Y_b)$

$$k_1 \cdot \begin{pmatrix}1\\1\end{pmatrix} + k_2 \cdot \begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}Y_a\\Y_b\end{pmatrix} \rightarrow \begin{matrix} k_1 + k_2 = Y_a \rightarrow k_2 = Y_a - Y_b. \\ k_1 = Y_b \end{matrix}$$

→ solución PVI:

$$\boxed{Y(t) = Y_b \cdot \begin{pmatrix}\cos t - \text{sen}\,t\\ \cos t\end{pmatrix} + (Y_a - Y_b) \cdot \begin{pmatrix}\cos t + \text{sen}\,t\\ \text{sen}\,t\end{pmatrix}}$$